

Heat and Mass Transfer Effect on Steady MHD Flow through an Isothermal Vertical Porous Surface

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Abstract - Magneto hydro dynamic effects of heat and mass transfer on continuously moving isothermal vertical porous surface is investigated. The boundary layer equations are transformed into ordinary differential equations. The Velocity, temperature and concentration profiles are studied for different non dimensional parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, porosity parameter and Hartmann number. The results are discussed with the help of graphs.

Keywords : MHD, heat transfer, mass transfer, temperature field, velocity field, concentration field, skin friction.

I. INTRODUCTION

The study of boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering process. To be more specific, heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass-fibre and paper production, the cooling of an infinite metallic plate in a cool bath, the boundary layer along a liquid film in condensation process, the material handling conveyors, cooling of metallic sheets or electronic chips, crystal growing etc.

There are many natural phenomena and engineering problems susceptible to magneto hydrodynamics. MHD finds practical uses in many areas such as pumping orientation and confinement of extremely hot ionized gases or plasmas in thermonuclear fusion experiments and space propulsion resulting from the electromagnetic acceleration of ionized gases. It is useful and important in Astrophysics as most part of the universe is filled widely spaced charged particles and permeates by magnetic fields and because the enormous scale of events makes up for the conductivity and magnetic fields. Magnetohydrodynamics finds applications in Ion Propulsion, electromagnetic pumps, MHD power generators, controlled fusion research, MHD couples and bearings, plasma jets and chemical synthesis, etc.

Sakiadis [1,2] studied the growth of the two dimensional velocity boundary layer over a continuously moving flat plate, emerging from a wide slot, at uniform velocity. The governing equations are solved using a similarity transformation. Tsou al [3] analysed the hydrodynamics stability of the flow within the framework of small –

perturbation method. The characteristics of the laminar boundary layer on continuous moving surfaces were described and an experiment was performed to demonstrate that such a flow was physically realizable. Thermal boundary layer on a continuously moving semi-infinite horizontal plate in the presence of transverse magnetic field with heat flux was studied by Murthy [4].

The similarity solutions and the resulting equations were integrated numerically. Varavelu [5] studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving, horizontal flat surface with uniform suction and internal heat generation/absorption. In all these studies, the authors have taken the continuous moving surface to be oriented in the horizontal direction. Again, Vajravelu [6] extended the problem of [5] to vertical surface. The heating as well as cooling effect of moving isothermal vertical plate were analyzed.

Heat transfer in a fluid over a linearly stretching sheet with variable thermal conductivity and internal heat generation has studied by K.Rajagopal et. al (2005). Hassan A.M.El-Arabawy obtained exact solutions of Mass transfer over a stretching surface with chemical reaction and suction/injection.

However, the theoretical solution for hydromagnetic convection on continuously moving isothermal vertical surface with uniform suction and mass diffusion is not studied in the literature. The present study deals with heat and mass transfer effects on flow past an impulsively started vertical surface in the presence of magnetic field. The effect of velocity for different parameters like thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and Hartmann number are studied graphically.

II. ANALYSIS

Consider a two-dimensional steady incompressible flow of a viscous fluid on a continuous vertical porous surface, issuing from a slot and moving with a uniform velocity u_w , in a fluid at rest, in the presence of a transverse magnetic field of strength B_0 . Let the x-axis be taken along the

direction of motion of the sheet in the upward direction and the y-axis is taken normal to it. The surface temperature and concentration level near the surface are raised uniformly. If

σ is the electrical conductivity of the fluid, then the governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2 u}{\rho} - \frac{v}{k} u \quad (2)$$

$$\rho C_p \left(u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} \right) = k \frac{\partial^2 T'}{\partial y^2} \quad (3)$$

$$u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} \quad (4)$$

The initial and boundary conditions are

$$\begin{aligned} u = u_w, v = -v_o = \text{const.} < 0, T' = T'_w, C' = C'_w \quad \text{at} \quad y = 0 \\ u \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned} \quad (5)$$

Making use of the assumptions that the velocity, temperature and concentration fields are independent of the distance parallel to the surface and the Boussinesq's approximation, equations (1) to (4) and the boundary conditions (5) can be written as

$$-v_o \frac{du}{dy} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + v \frac{d^2 u}{dy^2} - \frac{\sigma B_o^2}{\rho} u - \frac{v}{k} u \quad (6)$$

$$-\sigma C_p v_o \frac{dT'}{dy} = k \frac{d^2 T'}{dy^2} \quad (7)$$

$$-v_o \frac{dC'}{dy} = D \frac{d^2 C'}{dy^2} \quad (8)$$

and the corresponding initial and boundary conditions are

$$\begin{aligned} u = u_w, \quad T' = T'_w, C' = C'_w \quad \text{at} \quad y = 0 \\ u \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned} \quad (9)$$

On introducing the following non-dimensional quantities:

$$\begin{aligned} Y = \frac{yv_o}{v}, U = \frac{u}{u_w}, Gr = \frac{vg\beta(T'_w - T'_\infty)}{u_w v_o^2}, Gc = \frac{vg\beta^*(C'_w - C'_\infty)}{u_w v_o^2}, \\ T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Pr = \frac{\mu C_p}{k}, M = \frac{\sigma B_o^2 v}{\rho v_o^2}, Sc = \frac{v}{D}, K = \frac{v_o^2 k}{v^2} \end{aligned} \quad (10)$$

Equations (6) to (8) are reduced to the following non-dimensional form

$$\frac{d^2 U}{dY^2} + \frac{dU}{dY} + GrT + GcC - (M + \frac{1}{K})U = 0 \quad (11)$$

$$\frac{d^2 T}{dY^2} + Pr \frac{dT}{dY} = 0 \quad (12)$$

$$\frac{d^2C}{dY^2} + Sc \frac{dC}{dY} = 0 \quad (13)$$

and the corresponding initial and boundary conditions in non-dimensional form are

$$\begin{aligned} U = 1, T = 1, C = 1 \quad \text{at} \quad Y = 0 \\ U \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \quad \text{as} \quad Y \rightarrow \infty \end{aligned} \quad (14)$$

Solving equations (11) to (13) with boundary conditions (14), we get

$$U(Y) = \left[1 + \frac{Gr}{Pr^2 - Pr - M} + \frac{Gc}{Sc^2 - Sc - M} \right] \exp \left[-\frac{1}{2} \left(1 + \sqrt{1 + 4 \left(M + \frac{1}{K} \right)} \right) Y \right] - \left[\frac{Gr}{Pr^2 - Pr - M} \right] \exp(-Pr Y) - \left[\frac{Gc}{Sc^2 - Sc - M} \right] \exp(-Sc Y) \quad (15)$$

$$T(Y) = \exp(-Pr Y) \quad (16)$$

$$C(Y) = \exp(-Sc Y) \quad (17)$$

The dimensionless skin-friction at the surface are given by

$$\tau = \left(\frac{dU}{dY} \right)_{Y=0}$$

$$\tau = \frac{1}{2} \left[1 + \frac{Gr}{Pr^2 - Pr - M} + \frac{Gc}{Sc^2 - Sc - M} \right] \left[1 + \sqrt{1 + 4 \left(M + \frac{1}{K} \right)} \right] + \frac{Gr}{Pr^2 - Pr - M} + \frac{Gc}{Sc^2 - Sc - M}$$

III. RESULTS AND DISCUSSION

The computed solutions for the velocity, temperature, concentration and skin-friction are valid at some distance from the slot, even though suction is applied from the slot onward. Making use of the assumption that the velocity is independent of the distance parallel to the surface. The fluids considered in this study are air (Pr=0.71).

Figure1. shows the velocity profile for various thermal and mass Grashof number. From the profile, it is clear that the velocity profile increases with the increase of Gr and Gc.

Figure2. gives the velocity profile for different Prandtl number and Hartmann number. The profile decreases with the increase of Prandtl number and Hartmann number.

Figure3. exhibits the velocity profile for various Schmidt number and Porosity parameter. It is clear from the graph that with the increase in Schmidt number the profile decreases and the increase of porosity parameter increases the velocity profile.

Figure 4. shows that with the increase in Prandtl number the temperature profile decreases.

Figure5. exhibits the concentration profile for various Schmidt number. It is clear that increase in Schmidt number decrease the profile of concentration.

Figure6. shows the skin-friction profile for various Pr and M. It gives the idea that the profile increases with increase of Prandtl number and decreases with the increase of Hartmann number.

Figure7. exhibits skin-friction profile for various Sc and K. it is known that increase in Schmidt number and Porosity parameter decreases the profile. Figure8. shows the skin-friction profile for various Gr and Sc. It is clear that profile decreases with the increase of thermal Grashof number and Mass Grashof number.

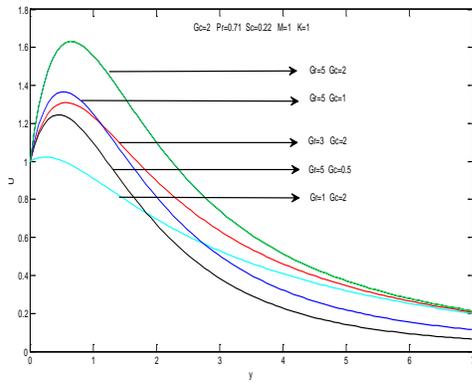


Fig.1 Velocity Profiles for different Gr and Gc

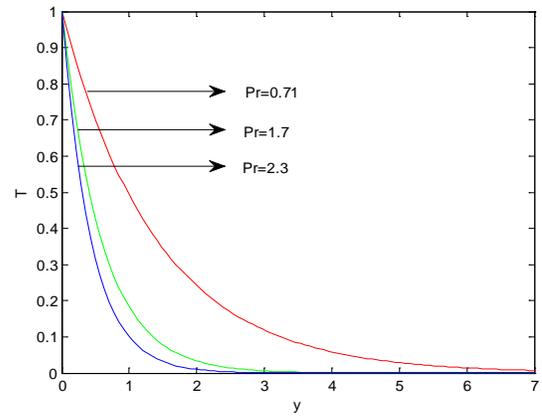


Fig.4 Temperature Profiles for different Pr

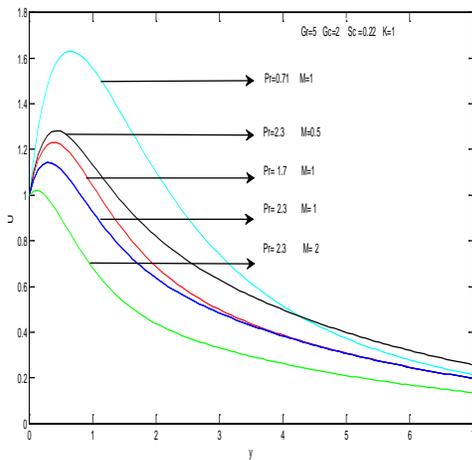


Fig.2 Velocity Profiles for different Pr and M

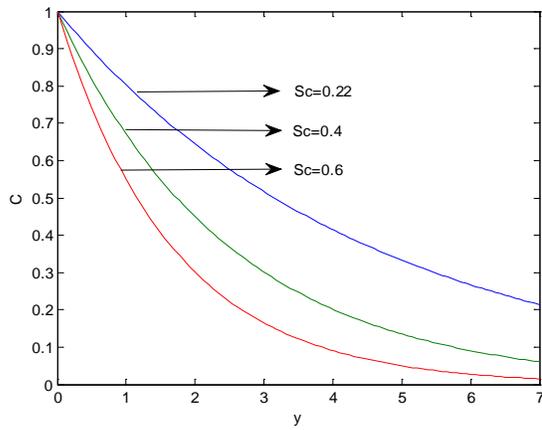


Fig.5 Concentration Profiles for different Sc

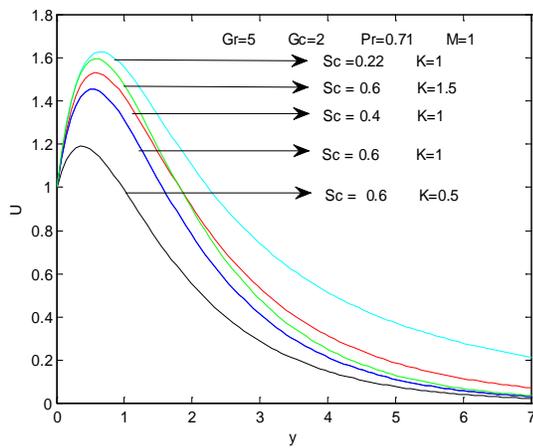


Fig.3 Velocity Profile for different Sc and K

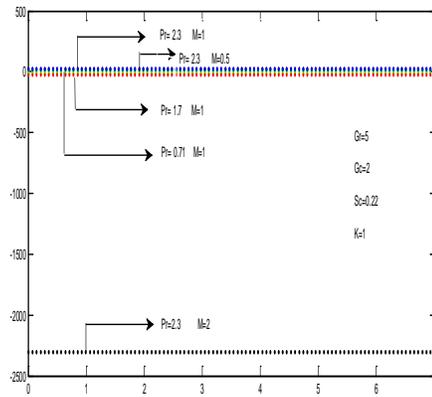


Fig.6 Skin-friction for different Pr and M

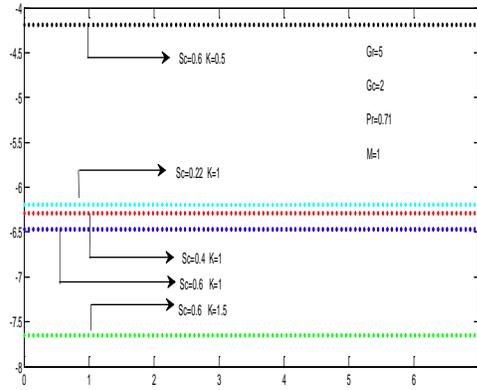


Fig.7 Skin-friction for different Sc and K

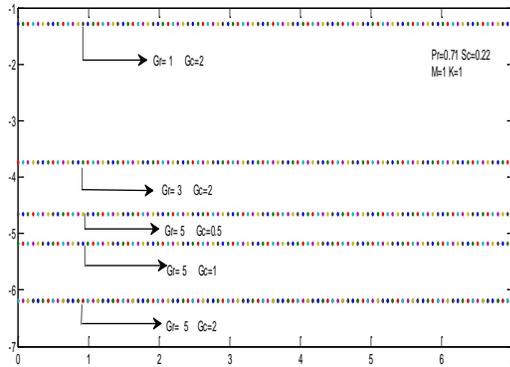


Fig.8 Skin-friction for different Gr and Gc

IV. CONCLUSIONS

Theoretical solution for Heat and mass transfer effects on flow past an impulsively started isothermal vertical porous surface in the presence of magnetic field with uniform suction is obtained. The effects of different Grashof number, Prandtl number and Hartmann number, Schmidt number and Porosity parameter are studied in detail. The solutions are in terms of exponential functions.

The study concludes the following results:

1. Velocity increases with increasing thermal Grashof number, mass Grashof number and Porosity parameter. The trend is just reversed with respect to Schmidt number, Prandtl number and Hartmann number.
2. Temperature decreases with the increase of Prandtl number.
3. Concentration decreases with the increase of Schmidt number.
4. There is a fall in the skin-friction due to the increase of Hartmann number, Schmidt number, Porosity parameter, thermal Grashof number and mass Grashof number. The profile increases with increase of Prandtl number.

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